DC Motor Calculations

Calculating Mechanical Power Requirements

In dc motors, electrical power (Pel) is converted to mechanical power (Pmech). In addition to frictional losses, there are power losses in Joules/sec (Iron losses in coreless dc motors are negligible).

$$Pel = Pmech + Pj\ loss$$

Physically, power is defined as the rate of doing work. For linear motion, power is the product of force multiplied by the distance per unit time. In the case of rotational motion, the analogous calculation for power is the product of torque multiplied by the rotational distance per unit time.

$$Prot = M \times \omega$$

Where:

Prot = rotational mechanical power
M = torque
ω = angular velocity

The most commonly used unit for angular velocity is rev/min (RPM). In calculating rotational power, it is necessary to convert the velocity to units of rad/sec. This is accomplished by simply multiplying the velocity in RPM by the constant $(2 \times \prod)/60$:
$\omega rad = \omega rpm \times (2\prod)/60$

It is important to consider the units involved when making the power calculation. A reference that provides conversion tables is very helpful for this purpose. Such a reference is used to convert the torque-speed product to units of power (Watts). Conversion factors for commonly used torque and speed units are given in the following table. These factors include the conversion from RPM to rad/sec where applicable.

Torque Units	Units Speed	Conversion Factor
oz-in	RPM	0.00074
oz-in	rad/sec	0.0071
in-lb	RPM	0.0018
in-lb	rad/sec	0.1130
ft-lb	RPM	0.1420
ft-lb	rad/sec	1.3558
N-m	RPM	0.1047

For example, assume that it is necessary to determine the power required to drive a torque load of 3 oz-in at a speed of 500 RPM. The product of the torque, speed, and the appropriate conversion factor from the table is:

$$3\text{oz-in} \times 500\text{rpm} \times 0.00074 = 1.11 \text{ Watts}$$

Calculation of power requirements is often used as a preliminary step in motor or gearmotor selection. If the mechanical power required for a given application is known, then the maximum or continuous power ratings for various motors can be examined to determine which motors are possible candidates for use in the application.

Torque - Speed Curves

One commonly used method of displaying motor characteristics graphically is the use of torque – speed curves. While the use of torque - speed curves is much more common in technical literature for larger DC machines than it is for small, ironless core devices, the technique is applicable in either case. Torque – speed curves are generated by plotting motor speed, armature current, mechanical output power, and efficiency as functions of the motor torque. The following discussion will describe the construction of a set of torque – speed curves for a typical DC motor from a series of raw data measurements. Motor 1624009S is used as an example.

Assume that we have a small motor that we know has a nominal voltage of 9 volts. With a few fundamental pieces of laboratory equipment, the torque - speed curves for the motor can be generated:

Step One (*measure basic parameters*):

Using a voltage supply set to 9 volts, run the motor unloaded and measure the rotational speed using a non-contacting tachometer (strobe, for instance). Measure the motor current under this no-load condition. A current probe is ideal for this measurement since it does not add resistance in series with the operating motor. Using an adjustable torque load such as a small particle brake coupled to the motor shaft, increase the torque load to the motor just to the point where stall occurs. At stall, measure the torque from the brake and the motor current. For the sake of this discussion, assume that the coupling adds no load to the motor and that the load from the brake does not include unknown frictional components. It is also useful at this point to measure the terminal resistance of the motor. Measure the resistance by contacting the motor terminals. Then spin the motor shaft and take another measurement. The measurements should be very close in value. Continue to spin the shaft and take at least three measurements. This will ensure that the measurements were not taken at a point of minimum contact on the commutator.

Now we have measured the:

- n0= no-load speed
- I0= no-load current
- MH= stall torque
- R= terminal resistance

Step Two (*plot current vs. torque and speed vs torque*):

Prepare a graph with motor torque on the horizontal axis, motor speed on the left vertical axis, and motor current on the right vertical axis. Scale the axes based on the measurements in step 1. Draw a straight line from the left origin of the graph (zero torque and zero current) to the stall current on the right vertical axis (stall torque and stall current). This line represents a plot of the motor current as a function of the motor torque. The slope of this line is the proportionality constant for the relationship between motor current and motor torque (in units of current per unit torque). The reciprocal of this slope is the torque constant of the motor (in units of torque per unit current). For the resulting curves see Graph 1.

For the purpose of this discussion, it will be assumed that the motor has no internal friction. In practice, the motor friction torque is determined using the torque constant of the motor and the measured no-load current. The torque vs speed line and the torque vs current line are then started not at the left vertical axis but at an offset on the horizontal axis equal to the calculated friction torque.
Step Three (plot power vs torque and efficiency vs torque):
In most cases, two additional vertical axes are added for plotting power and efficiency as functions of torque. A second left vertical axis is usually used for efficiency and a second right vertical axis is used for power. For the sake of simplifying this discussion, efficiency vs. torque and power vs. torque will be plotted on a second graph separate from the speed vs. torque and current vs. torque plots.
Construct a table of the motor mechanical power at various points from no-load to stall torque.
Since mechanical power output is simply the product of torque and speed with a correction factor for units (see section on calculating mechanical power requirements), power can be calculated using the previously plotted line for speed vs. torque. A sample table of calculations for motor M2232U12G is shown in Table 1. Each calculated point is then plotted. The resulting curve is a parabolic curve as shown in Graph 1. The maximum mechanical power occurs at approximately one-half of the stall torque. The speed at this point is approximately one-half of the no-load speed.

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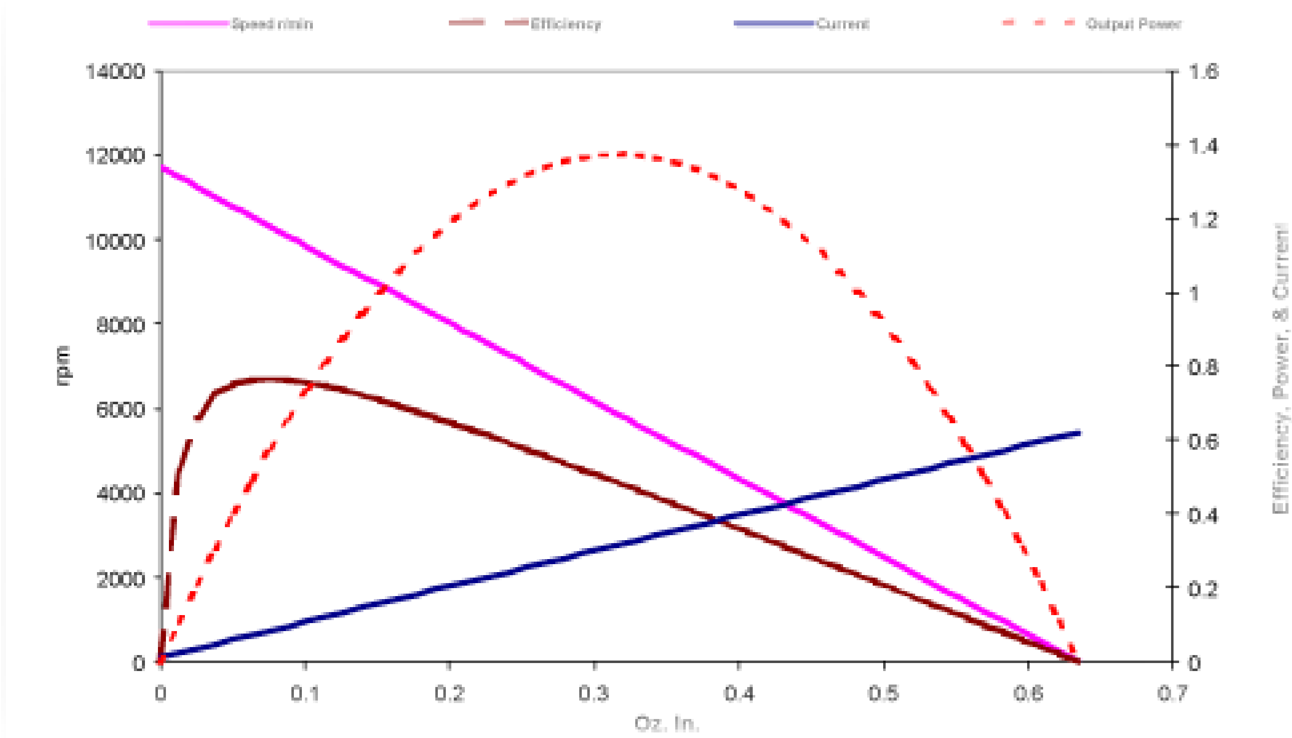
Construct a table of the motor efficiency at various points from no-load to stall torque. The voltage applied to the motor is given, and the current at various levels of torque has been plotted. The product of the motor current and the applied voltage is the power input to the motor. At each point selected for calculation, the efficiency of the motor is the mechanical power output divided by the electrical power input. Once again, a sample table for motor M2232U12G is shown in Table 1. and a sample curve in Graph 1. Maximum efficiency occurs at about 10% of the motor stall torque.

Torque (oz-in)	Speed (rpm)	Current (mA)	Power (Watts)	Efficiency (%)
0.025	11,247.65	0.024	0.208	0.10
0.05	10,786.3	0048	0.399	71.87
0.075	10,324.85	0.072	0.573	75.27
0.1	9,863.6	0.096	0.730	74.99
0.125	9,402.25	0.120	0.870	73.25
0.15	8,940.9	0.144	0.992	70.78
0.75	8,479.55	0.168	1.098	67.89
0.2	8,018.2	0.192	1.187	64.73
0.225	7,556.85	0.217	1.258	61.40
0.25	7,095.5	0.241	1.313	57.95
0.275	6,634.15	0.265	1.350	54.41
0.3	6,172.8	0.289	1.370	50.80
0.325	5,711.45	0.313	1.374	47.14
0.325	5,711.45	0.337	1.360	43.44
0.35	5,250.1	0.337	1.360	43.44
0.375	4,788.75	0.361	1.329	39.71
0.4	4,327.4	0.385	1.281	35.95
0.425	3,866.05	0.409	1.216	32.17
0.45	3,404.7	0.433	1.134	28.37
0.475	2,943.35	0.457	1.035	24.56
0.5	2,482	0.481	0.918	20.74
0.525	2020.65	0.505	0.785	16.90
0.55	1,559.3	0.529	0.635	13.05
0.575	1,097.95	0.577	0.283	5.34
0.6	636.6	0.577	0.283	5.34
0.625	175.25	0.602	0.081	1.47

Graph1



Numerical Calculation

For an iron-less core, DC motor of relatively small size, the relationships that govern the behavior of the motor in various circumstances can be derived from physical laws and characteristics of the motors themselves. Kirchoff's voltage rule states, "The sum of the potential increases in a circuit loop must equal the sum of the potential decreases." When applied to a DC motor connected in series with a DC power source, Kirchoff's voltage rule can be expressed as "The nominal supply voltage from the power source must be equal in magnitude to the sum of the voltage drop across the resistance of the armature windings and the back EMF generated by the motor.":

$$V0 = (I \times R) + Ve$$

Where:
Vo = Power supply (Volts)
I = Current (A)
R = Terminal Resistance (Ohms)
Ve = Back EMF (Volts)

The back EMF generated by the motor is directly proportional to the angular velocity of the motor. The proportionality constant is the back EMF constant of the motor.

$$Ve = \omega \times Ke$$

Where:
ω= angular velocity of the motor
ke = back EMF constant of the motor

Therefore, by substitution:

$$Vo = (I \times R) + (\omega \times Ke)$$

The back EMF constant of the motor is usually specified by the motor manufacturer in volts/RPM or mV/RPM. In order to arrive at a meaningful value for the back EMF, it is necessary to specify the motor velocity in units compatible with the specified back EMF constant. The motor constant is a function of the coil design and the strength and direction of the flux lines in the air gap. Although it can be shown that the three motor constants normally specified (back EMF constant, torque constant, and velocity constant) are equal if the proper units are used, calculation is facilitated by the specification of three constants in the commonly accepted units. The torque produced by the rotor is directly proportional to the current in the armature windings. The proportionality constant is the torque constant of the motor.

$$Mo = I \times Km$$

Where:
Mo = torque developed at rotor
kM = motor torque constant
Substituting this relationship:

$$V = (M \times R)/Km + (\omega \times Ke)$$

The torque developed at the rotor is equal to the friction torque of the motor plus the resisting torque due to external mechanical loading:

$$M0 = Ml + Mf$$

Where:
Mf = motor friction torque
Ml = load torque

Assuming that a constant voltage is applied to the motor terminals, the motor velocity will be directly proportional to sum of the friction torque and the load torque. The constant of proportionality is the slope of the torque-speed curve and can be calculated by:

$$\Delta n/\Delta M = n0 / MH$$

Where:
MH = stall torque

n0= no-load speed

An alternative approach to deriving this value is to solve for velocity, n:

$$n = (V0/Ke) - M/(kM \times Ke)$$

Differentiating both sides with respect to M yields:

$$\Delta n/\Delta M = -R / (kM \times Ke)$$

Using dimensional analysis to check units, the result is:

$$\text{-Ohms/(oz-in/A)} \times \text{(V/RPM)} = \text{-Ohm-A-RPM/V-oz-in} = \text{-RPM/oz-in}$$

It is a negative value describing loss of velocity as a function of increased torsional load.

Sample Calculation

Motor 1624T009S is to be operated with 9 volts applied to the motor terminals. The torque load is 0.2 oz-in. Find the resulting motor speed, motor current, efficiency, and mechanical power output. From the motor data sheet, it can be seen that the no-load speed of the motor at 12 volts is 11,700 rpm. If the torque load is not coupled to the motor shaft, the motor would run at this speed.

The motor speed under load is simply the no-load speed less the reduction in speed due to the load. The proportionality constant for the relationship between motor speed and motor torque is the slope of the torque vs. speed curve, given by the motor no-load speed divided by the stall torque. In this example, the speed reduction caused by the 0.2 oz -in torque load is:

$$0.2 \text{ oz-in} \times (11{,}700 \text{ rpm}/.634 \text{ oz-in}) = -3{,}690 \text{ rpm}$$

The motor speed under load must then be:

$$11{,}700 \text{ rpm} - 3{,}690 \text{ rpm} = 8{,}010 \text{ rpm}$$

The motor current under load is the sum of the no-load current and the current resulting from the load. The proportionality constant relating current to torque load is the torque constant (kM), in this case, 1.039 oz -in/A. In this case, the load torque is 0.2 oz-in, and the current resulting from the load must be:

$$I = 0.2 \text{ oz-in} \times 1 \text{ amp}/1.039 \text{ oz -in} = 192 \text{ mA}$$

The total motor current must be the sum of this value and the motor no-load current. The data sheet lists the motor no-load current as 60 mA. Therefore, the total current is:

192 mA + 12 mA = 204 mA

The mechanical power output of the motor is simply the product of the motor speed and the torque load with a correction factor for units (if required). Therefore, the mechanical power output of the motor in this application is:

output power = 0.2 oz-in x 8,010 rpm x 0.00074 = 1.18 Watts

The mechanical power input to the motor is the product of the applied voltage and the total motor current in Amps. In this application:

input power = 9 volts x 0.203 A = 1.82Watts

Since efficiency is simply power out divided by power in, the efficiency in this application is:

efficiency = 1.18 Watts / 1.82 Watts = 0.65 = 65%

Thermal Calculations

A current I flowing through a resistance R results in a power loss as heat of I2R. In the case of a DC motor, the product of the square of the total motor current and the armature resistance is the power loss as heat in the armature windings. For example, if the total motor current was .203 A and the armature resistance 14.5 Ohms the power lost as heat in the windings is:

power loss = 0.2032 x 14.5 = 0.59 Watts

The heat resulting from I2R losses in the coil is dissipated by conduction through motor components and airflow in the air gap. The ease with which this heat can be dissipated is a function of the motor type and construction. Motor manufacturers typically provide an indication of the motor's ability to dissipate heat by providing thermal resistance values. Thermal resistance is a measure of the resistance to the passage of heat through a given thermal path. A large cross section aluminum plate would have a very low thermal resistance, for example, while the values for air or a vacuum would be considerably higher. In the case of DC motors, there is a thermal path from the motor windings to the motor case and a second between the motor case and the motor environment (ambient air, etc.). Some motor manufacturers specify a thermal resistance for each of the two thermal paths while others specify only the sum of the two as the total thermal resistance of the motor. Thermal resistance values are specified in temperature increase per unit power loss. The total I2R losses in the coil (the heat source) are multiplied by thermal resistances to determine the steady state armature temperature. The steady state temperature increase of the motor (T) is given by:

Tinc = I2R x (Rth1 + Rth2)

Where:

Tinc = temperature increase
I = current through motor windings
R = resistance of motor windings
Rh1 = thermal resistance from windings to case
Rh2 = thermal resistance case to ambient

For example, a 1624E009S motor running with a current of 0.203 Amps in the motor windings, with an armature resistance of 14.5 Ohms, a winding-to-case thermal resistance of 8 °C/Watt, and a case-to-ambient thermal resistance of 39 °C/Watt. The temperature increase of the windings is given by:

T = .2032 x 14.5 x (8 + 39) = 28°C

If it is assumed that the ambient air temperature is 22°C, then the final temperature of the motor windings is 50°C (22° + 28°).

It is important to be certain that the final temperature of the windings does not exceed their rated value. In the example given above, the maximum permissible winding temperature is 100°C. Since the calculated winding temperature is only 50°C, thermal damage to the motor windings will not be a problem in this application. One could use similar calculations to answer a different kind of question. For example, an application may require that a motor run at its maximum torque without being damaged by heating. To continue with the example given above, suppose it is desired to run motor 1624E009S at the maximum possible torque with an ambient air temperature of 22°C. The designer wants to know how much torque the motor can safely provide without overheating.

The data sheet for motor 1624E009S specifies a maximum winding temperature of 100°C. Since the ambient temperature is 22°C, a rotor temperature increase of 78°C is tolerable. The total thermal resistance for the motor is 47°C/Watt. By taking the reciprocal of the thermal resistance and multiplying this value by the acceptable temperature increase, the maximum power dissipation in the motor can be calculated:

P = 78° x 1 Watt/47° = 1.66 Watts

Setting I2R equal to the maximum power dissipation and solving for I yields the maximum continuous current allowable in the motor windings:

I2 = 2.19 Watts / 14.15 ohms
I2R = 2.19 Watts
I = .338 Amps

The motor has a torque constant of 1.86 oz-in/A and a no-load current of 60 mA. Therefore, the maximum current available to produce useful torque is .530 Amps (.590 - .060), and the maximum usable torque available (M) is given by:

$$M = .327\text{ A} \times 1.309\text{ oz-in/A} = 0.428\text{ oz-in}$$

The maximum allowable current through the motor windings could be increased by decreasing the thermal resistance of the motor. The rotor-to-case thermal resistance is primarily fixed by the motor design. The case-to-ambient thermal resistance can be decreased significantly by the addition of heat sinks. Motor thermal resistances for small DC motors are usually specified with the motor suspended in free air. Therefore, there is usually some heat sinking which results from simply mounting the motor into a framework or chassis. Some manufacturers of larger DC motors specify thermal resistance with the motor mounted into a metal plate of known dimensions and material.

The preceding discussion does not take into account the change in resistance of the copper windings as a result of heating. While this change in resistance is important for larger machines, it is usually not significant for small motors and is often ignored for the sake of calculation.